Reconstruction of interfaces using elliptic partial differential equations

Application to the geometric inverse problem of corrosion detection in a copper converter

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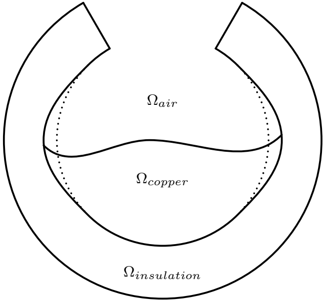
General objectives associated with the project

- Study theoretically/numerically the reconstruction of a damaged thermal insulating material in a copper converter using electrostatic/thermal measurements.
- Give some quantitative/qualitative results concerning the unknown shape of the interface air/copper and the location of the purified copper, especially near the insulating layer.

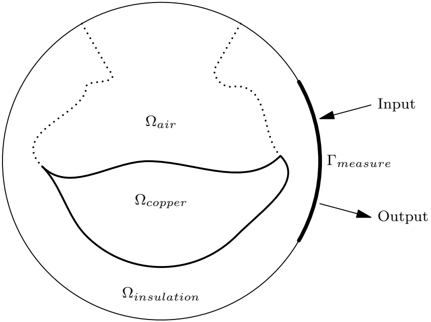


Recovering the conductivity from boundary measurements

The situation can be modelled by the following equation:

$$\begin{cases} \operatorname{div}(\gamma \nabla u) = 0 & \text{in} \quad \Omega := \Omega_{air} \sqcup \Omega_{copper} \sqcup \Omega_{insulation} \\ u = \varphi & \text{on} \quad \Gamma_{measure} \\ \partial_{\mathbf{n}} u = 0 & \text{on} \quad \partial\Omega \backslash \Gamma_{measure}. \end{cases}$$

Goal: recover $\gamma = \gamma_{air} \mathbf{1}_{\Omega_{air}} + \gamma_{copper} \mathbf{1}_{\Omega_{copper}} + \gamma_{insulating} \mathbf{1}_{\Omega_{insulating}}$ from the knowledge of $\partial_{\mathbf{n}} u|_{\Gamma_{measure}}$.

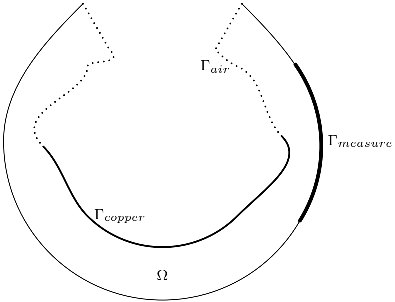


A geometric inverse problem as a limiting case

Since $\gamma_{air} = 0$ and $\gamma_{copper} = +\infty$, we obtain the boundary value problem:

$$\begin{cases} \Delta u = 0 & \text{in} \quad \Omega \\ u = \varphi & \text{on} \quad \Gamma_{measure} \\ u = 0 & \text{on} \quad \Gamma_{copper} \\ \partial_{\mathbf{n}} u = 0 & \text{on} \quad \Gamma_{air} \quad \text{and} \quad \partial\Omega \backslash \left(\Gamma_{measure} \sqcup \Gamma_{air} \sqcup \Gamma_{copper} \right). \end{cases}$$

Goal: recover Γ_{air} and Γ_{copper} from the knowledge of $\partial_{\mathbf{n}} u|_{\Gamma_{measure}}$, add some geometrical informations at the transition point (position, normal).

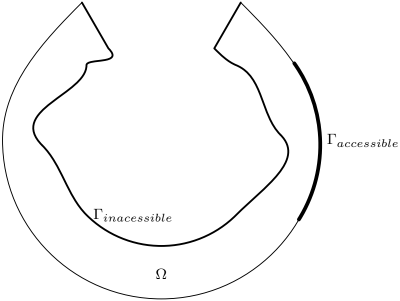


Another corrosion detection problem

The corrosion can also be modelled by the boundary value problem:

$$\begin{cases} \Delta u = 0 & \text{in} \quad \Omega \\ u = \varphi & \text{on} \quad \Gamma_{accessible} \\ \partial_{\mathbf{n}} u + \gamma u = 0 & \text{on} \quad \Gamma_{inacessible}, \\ \partial_{\mathbf{n}} u = 0 & \text{on} \quad \partial\Omega \backslash \left(\Gamma_{accessible} \sqcup \Gamma_{inaccessible}\right) \end{cases}$$

Goal: recover γ from the knowledge of $\partial_{\mathbf{n}} u|_{\Gamma_{measure}}$, locate the purified copper and the slag phase.



Some perspectives of research

- **Industrial issues:** using asymptotic analysis and perturbations methods, give some informations about the transition air/copper, slag/purified phases, damage width of the insulating layer.

- **Mathematical issues:** wellposedness of the geometric inverse problems (uniqueness), good stability estimates (Lipschitz).

- **Numerical issues:** these problems can also be seen as shape optimization ones by considering for example the functional:

$$\inf_{\Omega} \int_{\Gamma_{measure}} |\partial_{\mathbf{n}} u_{\Omega} - g|^2,$$

where u_{Ω} is the solution of a PDE posed on a domain Ω with φ as a fixed input and g as the fixed output to reach.