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Mini Workshop on Control and Inverse Problems of PDE's

January 18-19, 2016

Venue

Departamento de Matemática
Universidad Técnica Federico Santa María
Avenida España 1680, Valparaíso

Organizers: Nicolás Carreño (nicolas.carrenog@usm.cl), Eduardo Cerpa (eduardo.cerpa@usm.cl), Alberto Mercado (alberto.mercado@usm.cl), Axel Osses (axosses@dim.uchile.cl).

Speakers

1. Lucie Baudouin (LAAS-CNRS)
2. Maya de Buhan (Université Paris Descartes)
3. Felipe Chaves-Silva (Université Nice Sophia Antipolis)
4. Matías Courdurier (Pontificia Universidad Católica de Chile)
5. Anna Doubova (Universidad de Sevilla)
6. Sylvain Ervedoza (Université Paul Sabatier)
7. Mamadou Gueye (Universidad Técnica Federico Santa María)
8. Rodrigo Lecaros (Universidad de Chile)
9. Alberto Mercado (Universidad Técnica Federico Santa María)
10. Cristhian Montoya (Universidad de Chile)
11. Axel Osses (Universidad de Chile)
12. Enrique Otárola (Universidad Técnica Federico Santa María)
13. Ivonne Rivas (Universidad del Valle)

Program

	Monday 18	Tuesday 19
10:00-10:20		Baudouin
10:20-10:40		de Buhan
10:40-11:00		Courdurier
11:00-11:30	Coffee break	Coffee break
11:30-11:50	Ervedoza	Mercado
11:50-12:10	Lecaros	Rivas
12:10-12:30	Otárola	Osses
12:30-14:00	Lunch	Lunch
14:00-16:30	Working time	Working time
16:30-17:00	Coffee break	Coffee break
17:00-17:20	Doubova	Gueye
17:20-17:40	Chaves-Silva	Montoya
20:00-∞	Dinner	

Titles and abstracts

Open loop stabilization of the incompressible Navier-Stokes equation in a channel with a normal control

Sylvain Ervedoza

Abstract. In this talk, I will present an open loop stabilization result for the incompressible Navier-Stokes equation when the control acts only on the normal component of the solution. In this case, the linearized model is not controllable. We therefore have to use the non-linear terms in order to stabilize the missing directions. We do that by performing a power series expansion. This is a joint work with Shirshendu Chowdhury and Jean-Pierre Raymond.

Some soliton solutions of a $1D$ non-linear Schrödinger equation

Matías Courdurier

Abstract. In different applications of theoretical physics like non-linear optics, Bose-Einstein condensates, gravity waves and other non-linear phenomena, the non-linear Schrödinger equation plays a crucial role.

In this talk we will look at the non-linear Schrödinger equation

$$i\partial_t u = -\frac{1}{2}\partial_x^2 u + V(x,t,u) \qquad (1)$$

with specific time-space dependent non-linear term $V(x,t,u)$ and we will present a family of soliton solutions.

A FEM for an optimal control problem of fractional powers of elliptic operators

Enrique Otárola

Abstract. We study solution techniques for a linear-quadratic optimal control problem involving fractional powers of elliptic operators. These fractional operators can be realized as the Dirichlet-to-Neumann map for a nonuniformly elliptic problem posed on a semi-infinite cylinder in one more spatial dimension. Thus, we consider an equivalent formulation with a nonuniformly elliptic operator as the state equation. The rapid decay of the solution to this problem suggests a truncation that is suitable for numerical approximation. We discretize the proposed truncated state equation using first-degree tensor product finite elements on anisotropic meshes. For the control problem we analyze two approaches: one that is semidiscrete based on the so-called variational approach, where the control is not discretized, and the other one that is fully discrete via the discretization of the control by piecewise constant functions. For both approaches, we derive a priori error estimates with respect to degrees of freedom. Numerical experiments validate the derived error estimates and reveal a competitive performance of anisotropic over quasi-uniform refinement.

Numerical solution of some geometric inverse problems

Anna Doubova

Abstract. We consider some geometric inverse problems for the wave and the Lam equations motivated by Elastography. We present several recent results and open questions concerning the numerical reconstruction of the unknown domain where the equations evolve. In the numerical experiments, we solve appropriate optimization problems. This needs the numerical solution of the PDE's, that is performed with `FreeFem++`. The routines on the `ff-NLopt` package, that provide an interface to a free/open-source library for nonlinear optimization, are also required. We present some numerical results in the $2D$ and $3D$ cases. This is joint work with E. Fernndez-Cara (Dpto. E.D.A.N., Universidad de Sevilla).

Controllabity for Pseudoparabolic equations

Felipe Chaves-Silva

Abstract. We discuss the Controllability of a class of equations of Sobolev-Galpern type. In general, this type of equations are not controllable from a fixed control region. We use the idea of moving controls to study controlability properties for this kind of equations.

Reconstruction of the potential in the wave equation : Part I & Part II

Lucie Boudouin and Maya de Buhan

Abstract. These two talks will develop the theoretical and numerical study of a reconstruction algorithm of the potential in a wave equation from boundary measurements, using a cost functional built on weighed energy terms from a Carleman estimate. This inverse problem for the wave equation consists in the determination of an unknown time-independent potential from a single measurement of the Neumann derivative of the solution on a part of the boundary. While its uniqueness and stability properties are already well known and studied, a constructive and globally convergent algorithm based on Carleman estimates for the wave operator was recently proposed and the numerical implementation of this strategy still presents several challenges, that we propose also to address here.

Control of underwater vehicles in inviscid fluids
Rodrigo Lecaros
joint work with Lionel Rosier

Abstract. In this talk, we present the advances related with the study of the controllability of an underwater vehicle immersed in an infinite volume of an inviscid fluid in $\mathbb{R}^3$. Taking as control input the flow of the fluid through a part of the boundary of the rigid body.

In [1], We assumed that the flow is irrotational. And we obtained a finite-dimensional system similar to Kirchhoff laws in which the control input appears through both linear terms (with time derivative) and bilinear terms. Applying Coron's return method, we established some local controllability results for the position and velocities of the underwater vehicle. Examples with six, four, or only three controls inputs are given for a vehicle with an ellipsoidal shape.

In a recent work, the authors pursue this study, by considering the more general case of a flow with vorticity. It is shown that the controllability of the position and the velocity of the underwater vehicle (a vector in $\mathbb{R}^{12}$) holds in a flow with vorticity whenever it holds in a flow without vorticity.

[1] R. Lecaros and L. Rosier. Control of underwater vehicles in inviscid fluidsI. Irrotational flows. ESAIM Control Optim. Calc. Var., 20(3):662703, 2014.

Controllability of systems of Schrödinger equations
Alberto Mercado

Abstract. We present some control problems regarding systems of coupled partial differential equations, when one of them is a linear Schrödinger equation, and using less controls than equations. 1) A system with two N-dimensional linear Schrödinger equations with a control supported in a region not satisfying the classical geometrical control condition; and 2) a system coupling a Schrödinger and a linear KdV equation.

Boundary controllability of Korteweg-de Vries type systems

Ivonne Rivas
joint work with Ademir Pazoto

Abstract. We study the boundary controllability of a nonlinear coupled system of two Korteweg-de Vries equations posed on an interval. This model describes the interactions of two weakly nonlinear gravity waves in a channel. Due to the nature of the system six boundary conditions are required. However, to study the exact local controllability property, we consider different combination of control inputs, with a maximum of four of them. First, the results are obtained for the linearized system through the duality-compactness approach together with some hidden regularity properties of the boundary terms. Then, using the contraction mapping theorem we derive the local controllability for the full system. In both cases, the results depend on the spatial domain.

Local null controllability of the N-dimensional Navier-Stokes system with nonlinear Navier-slip boundary conditions and $N-1$ scalar controls

Cristhian Montoya

Some inverse source problems for systems with one missing component

Axel Osses

Abstract. I motivate the importance of this type of problems, and present some ideas how to afford them using Carleman inequalities for parabolic and hyperbolic type systems.